

mathematics, and in this volume deals only with arithmetic and algebra; but his treatment is thorough, and his aim has been to give exact references to the original authorities for the statements in the text. The amount of labour that this has involved must have been very great; when the work is complete, with the indexes promised by the author, it will be a valuable repertory for those who wish to learn the facts at first hand. The number of bibliographical footnotes exceeds 1200, and since many of these give more than one reference, it will be seen how great a service the author has rendered to those who are inclined for research.

But the book is far from being a mere dry collection of facts and references. The style is concise, and there is no catchpenny rhetoric, but there is plenty to interest any intelligent reader. The arrangement allows us to trace in detail the development of methods and of notation; we are shown, with explanations, the actual symbols used and the processes employed by our predecessors; most important of all, there is an appendix with a selection of original examples ranging from Alchwarizmi to Leibniz and Newton. Few things are more instructive than an inspection of some of the older methods in arithmetic. Until the end of the fifteenth century, long after the decimal notation and the use of the "Arabic" numerals had become familiar, and when arithmetical calculations were usually worked on paper, the rule for performing long division was of a most complicated character, with rows of figures above the dividend as well as below, and tedious cancellings and substitutions which must have made the operation both laborious and liable to error. It is almost certain that the process is of Indian origin, and it is probable that the figures which, in written examples, we find cancelled by a stroke drawn through them represent digits which were actually obliterated at an earlier period, when the calculation was performed with a stick on a layer of sand.

A striking feature of early European books on arithmetic is the bewildering number of their so-called "rules." One reason for this is simple enough. Many of these books were intended to help business men—bankers, merchants, and so on—in such calculations as their calling obliged them to do. Their interest in arithmetic was purely practical, and all they wanted was a bundle of recipes for getting correct answers to questions of certain special types. Even in our own day we occasionally see such terms as "agricultural book-keeping" or "chemical arithmetic," which show that a demand for this sort of thing is not yet extinct. But even in treatises of a more theoretical kind *duplatio* and *mediatio*, in other words doubling and halving, were reckoned as separate rules. This is a historical survival, a sort of fossil relic of prehistoric times. It appears that the ancient Egyptians performed multiplication by a process practically equivalent to converting the multiplier into the binary scale; thus

$$x \times 13 = x \times 8 + x \times 4 + x,$$

where  $x \times 8$  and  $x \times 4$  were obtained by successive doubling. When an improved method of multiplication had been discovered, the older process became

obsolete; but *duplatio* held its ground as a special rule, in recognition, so to speak, of its former importance.

A considerable portion of this volume is naturally devoted to the theory of surds, and this cannot be separated from the Greek theory of geometrical irrationals. After all that has been written on the subject, *lacunae* remain which will probably never be filled up, unless new documents are discovered. Some undoubted facts are very puzzling when taken in combination. For instance, Euclid says in so many words that incommensurable quantities are not related to each other as numbers, and it really does seem that to a Greek geometer of Euclid's time the relation, as to length, of the diagonal of a square to one side was something different *in kind* from the relation of two commensurable distances. At the same time the Greeks must have been practically acquainted with what we should call rational approximations to  $\sqrt{2}$ , and it is well known that the irrationalities considered in the tenth book of Euclid's "Elements," when put into an algebraic form, correspond exactly to all the members of a particular group of surds, without omission or redundancy. Did the geometers, who professed to despise "logistic" in public, privately make use of it to help them in their researches?

Other subjects considered under the head of algebra are the development of the idea of number in general, the operations of algebra and their symbols, proportion, and equations. Under the last heading Diophantine analysis is included, and it may be noted as a fact not generally known that Diophantine equations of the form

$$px^2 - qy^2 = r$$

were actually discussed in India at least as early as the time of Brahmagupta—that is to say, more than a thousand years before Fermat proposed the Pellian equation to the English mathematicians. G. B. M.

#### OUR BOOK SHELF.

*La Lutte pour l'Existence et l'Évolution des Sociétés.*  
By J. L. de Lanessan. Pp. 277. (Paris: Félix Alcan, 1903.) Price 6 francs.

THE title of this book is most misleading. The reader naturally expects to find an account of the struggle for existence among primitive men and of the evolution which has resulted from the struggle. The first chapter has quotations from Buffon and Darwin which leave no doubt in one's mind that this is the line which is to be followed. After this comes a description of primitive society or rather the social system which the author assumes to be primitive. The struggle for existence drops out, and is not mentioned. Society begins, he tells us, with a severely patriarchal *régime*. He seems not to have heard of an earlier polyandrous period. Out of the family bond arose the sense of duty. Speaking of the tribe, he lays it down that the chieftain was regarded as the owner of all the land which the tribe possessed.

After this glance at primitive society, we plunge into French history. Many great questions are dealt with, and most of them with remarkable shrewdness. Our author discusses the origin of feudalism. He next decides that Christianity had nothing to do with the abolition of slavery. He traces the growth of the idea of liberty among the peasantry; it showed itself

in the *jacquerie* of the fifteenth century, which was the precursor of the revolution. He has much to say about the power of the clergy, about the national attitude of the Gallican Church followed by its eventual submission to the Pope.

With the revolution we begin to get a glimpse of what was in the author's mind when he gave his book its title. Only for a short time do the plebeians obtain liberty. Power is soon grasped once more by the well-to-do classes. What the revolution gained for the working man was the right to work when and at what work he chose. It introduced, in fact, free competition among individuals. As an individual the workman was free, but associations of workmen had as yet no legal status. Indeed, in 1791 the national assembly forbade combinations, whether among workmen or employers, intended to influence wages or prices. It is against this free competition between individuals that M. de Lanessan preaches. So far from bringing the best and strongest to the top, this Darwinian struggle for existence, as he terms it, causes nothing but misery. But has he any understanding of Darwinism? Darwin recognised not only a struggle between individuals, but a struggle between groups. If France is to hold her own against rivals, there must, no doubt, be mutual help among her citizens. There is nothing un-Darwinian in this. At the opening of the last part of the book, our author gives his views upon heredity, and it turns out that he is so Lamarckian that the struggle for existence seems to find no place in his theory of evolution; it is only a pest to be put an end to. After this, he passes on to the subject of the amount of food required by a workman, thence to alcoholism, which he attributes mainly to want of proper food, thence to the injurious effect of many of the substances employed in manufactures, thence to factory legislation. With all the main evils from which the workman suffers, the Third Republic has made an honest attempt to grapple. Before 1870 the policy of *laissez faire* was in the ascendant. There is much of interest in the book, and the style is clear, but "The Duties of the State" would have been a better title.

F. W. H.

*Ore Deposits. A Discussion.* Pp. 90. (New York: Engineering and Mining Journal, 1903.) Price 5s. net.

GEOLOGISTS and miners will be grateful to Mr. T. A. Rickard, the editor of the *Engineering and Mining Journal*, for having reprinted the report of a discussion upon ore deposits which took place before the Geological Society of Washington in the early part of the present year. Many leading American geologists, whose names are identified with the study of mineral deposits, were present, and took part in the discussion, so that the mining engineer now has before him, in the form of a small handy volume, a clear and authoritative statement of the views of men well qualified to express opinions upon a very difficult subject.

Geologists are accustomed to frame hypotheses upon the origin of rocks, and naturally they are dissatisfied with a classification of ore deposits dependent upon form, and favour genesis as a basis of arrangement. In this spirit Mr. W. H. Weed put forward his tentative classification of ore deposits, which occupies two and a half pages of the book. He wisely admits that ore deposits may have originated in very many ways, and says that his six classes "have been arranged to show gradation from the magmatic segregation of original igneous rocks to the deposits directly or indirectly due to the emanations from igneous rocks up to those due entirely to aqueous agencies."

Mr. J. E. Spurr followed with another classification,

NO. 1778, VOL. 69]

and Mr. C. R. Van Hise with a third. "Who shall decide when doctors disagree?" Until geologists are in harmony among themselves, the humble miner will probably do well to wait, much as he would like to have a purely genetic classification, and rest content in the meantime with his old subdivisions according to form. Besides, the miner wants something broader than a mere classification of ore deposits; he has to deal not only with ores, but also with the so-called "non-metallic" minerals, such as abrasives, borax, diamonds, gypsum, petroleum, phosphates, &c., and he consequently desires a scheme of arrangement of all mineral deposits less narrow than will be found in a treatise upon "Erzlagerstättenlehre."

*Storage Battery Engineering.* By Lamar Lyndon, B.E., M.E. Pp. viii+382. (New York: McGraw Publishing Co., 1903.) Price 3 dollars.

THIS book aims chiefly at treating the engineering side of storage batteries, such as the design and installation of a battery equipment, the precautions which have to be taken to maintain such an equipment in good working order, and the various accessory devices which have to be used therewith. The chemical side of the subject is treated very briefly; the first chapter, of less than a dozen pages, is all that is allotted to general theory. In the remaining chapters of the first part the characteristics of lead cells are considered in detail; the leading types of cell are described, and there is the usual series of illustrations of different grids. Considering that the book makes no pretence of being a complete treatise on accumulators, we think that much of the matter here included might with advantage have been omitted, and the material sifted with more discrimination. There are also several instances of carelessness; for example, the author speaks of forming Planté plates in a solution of litharge in *potassium*, a mistake repeated three or four times in a couple of pages. The treatment of the electrical and mechanical sides is less open to objection, and many useful suggestions are given as the results of actual experience.

The second part of the book is devoted to auxiliary apparatus; it is concerned with the use of accumulators in connection with distribution systems. The author describes at length the use of end cells or counter E.M.F. cells for voltage regulation, and the most suitable types of switches, hand regulated and automatic, to employ with them. The use of boosters and methods of wiring are considered at considerable length. On the whole the book should prove of value to the practical engineer, as it deals with an aspect of the storage battery which has not hitherto, so far as we are aware, received much systematic consideration.

M. S.

*Cassell's Popular Science.* Edited by Alexander S. Galt. Pp. viii+576. (London: Cassell and Co., Ltd., 1903.) Price 12s.

THIS handsome volume is a worthy attempt to popularise the physical, chemical, biological, and geological sciences. As the editor remarks, popular science has too often been synonymous with inexact science, and any attempt to show that scientific knowledge may be presented in an interesting manner, and be at the same time correct, is to be welcomed. The book is profusely illustrated and contains a well-selected series of brightly written essays on various subjects of pure and applied science. The volume may be recommended as a suitable present to boys and girls, who will probably by its means be led to study more deeply one of the many branches of science of which some of the methods and results are described.